COMPUTER-AIDED SYNTHESIS OF LOSSY COMMENSURATE LINE NETWORK AND ITS APPLICATION

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ABSTRACT

In this paper, a useful theorem which extends the technique introduced in [1] to even wide fields is proposed for transformation between distributed lossy and lumped lossless networks. A computer-aided approach is developed to the synthesis of lossy commensurate line network with all lines being of frequency dependent loss. As an application, one-stage broad-band amplifier is designed for monolithic microwave integrated circuits(MMIC's) and its performance is compared with the example in [2].

I. INTRODUCTION

It is well-known that MMIC's are having wide applications in radar, spaceflight, satellite and military communications, etc. Matching network is one of the important parts in MMIC's and is usually constructed by lumped and distributed elements. These matching elements, which are fabricated on GaAs semi-insulating substracts, have losses much more than those of matching elements developed in conventional hybrid integrated circuits, so that the presently efficient techniques[2-9] for synthesizing lumped and distributed lossless matching networks are not suitable for the lossy matching networks. In accordance with this problem, the authors in [1] introduced a theorem for transformation between lumped lossy or lossless network and lossless one, and developed a new computer-aided technique for treating the synthesis of lumped matching network with arbitrary nonuniform losses. However, it is clear to see that only a smaller part of the problem has been solved by now. As frequency increasing to millimeter wave range, the advantages of employing the lumped network, such as occupation of less GaAs wafer, broader bandwidth capability, and so on, will be indistinctive. In reverse, the distributed network appears to be superior to the lumped one in many asperts. For example, transmission lines have less paracitic reactances and their characteristic impedances can be realized easily and exactly.

The modern design of microwave TEM distributed networks is based upon a complex plane transformation introduced by Richards in 1948[10]. He showed that by a suitable frequency transformation, distributed networks, which are composed of commensurate lengths of transmission line and lumped resistors, could be treated in analysis or synthesis as *lumped lossless networks*(abbreviated as LLLN). Laterly, lots of authors were stimulated by his article and great achievements were obtained. As for the synthesis of distributed lossy network, to our knowledge, few of published papers dealt with it. The reason why people didn't concern with the losses in the synthesis and/or design of the distributed network, is that i) for a great number of practical problems, these losses are too small in comparison with the losses affected by others such as discontinuity. But for MMIC's, losses in transmission lines are not negligible; ii) the Richards' correspondence is limited to the lossless case [10], even though it is very useful for the synthesis of lossless commensurate line network. By now, people have not found a simple and effective method to obtain the parameters of the distributed lossy network.

In order to consider the losses of the transmission lines in the synthesis of a distributed networks, the authors introduce a new and useful theorem which extends the technique described in [1] to even wide fields and can obtain the parameters of a corresponding *lossy commensurate line network*(simply called LYCLN) from those of a supposed LLLN. It will be seen clearly in comparison with the result shown in [2] that the new method is practically applicable, considerably simplified, and can yield any complex models of the commensurate lines with arbitrary frequency dependent losses. Furthermore, a computer-aided method is presented to show the detailed synthesis steps of a broad-band monolithic microwave integrated FET amplifier with LYCLNs as matching networks.

II. TRANSFORMATION OF THE LOSSY COMMENSURATE LINE NETWORK

Richards [10], in his famous paper, first used the following transformation

$$\lambda=\tanh[\gamma(s)\tau]=\frac{\exp[\gamma(s)\tau]-\exp[-\gamma(s)\tau]}{\exp[\gamma(s)\tau]+\exp[-\gamma(s)\tau]} \tag{1}$$

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to synthesize a lossless commensurate line network, where $\gamma(s)=s$ is the propagation constant and τ the delay($\tau=L/cvp$, L is the fixed line length, $c_{vp}=3/10^{11}$ mm/sec the velocity of propagation on the line) of all the transmission line elements. By this correspondence, a complex angular frequency in s plane can be mapped into λ plane and many theorems for LLLNs can be "translated" into theorems about the lossless commensurate line networks. Thereafter, Richards' transformation became a theoretical basis for almost all of the published papers about the distributed network synthesis, and the relevant theorems were used to treat the lossless commensurate line networks. However, a general method has not yet been found by now for the synthesis of LYCLN with all lines having frequency dependent propagation constants and characteristic impedances.

In general, the propagation constant and characteristic impedance of a lossy transmission line are of frequency dependent function and can be written as

$$\gamma(s)=\sqrt{(s+d_L)(s+d_C)} \tag{2a}$$

$$Zo(s)=Zo_t\delta(s) \tag{2b}$$

and

$$\delta(s)=\sqrt{(s+d_L)/(s+d_C)} \tag{3a}$$

$$Zo_t=\sqrt{l/c} \tag{3b}$$

where $d_L=r/l$ and $d_C=g/c$; r, l, g, and c are the series resistance, series inductance, shunt conductance, and shunt capacitance, all per unit length, for a given line. From (2b) and (3), it can be found that the characteristic impedance Zo(s) may be divided into two parts, one is the frequency dependent function $\delta(s)$, and the other is the real positive multiplicative constant Zo_t. If the lossy transmission line reduces to a corresponding lossless one, $\delta(s)$ will be equal to 1 and Zo(s) to Zo_t, which is usually called characteristic impedance of the lossless transmission line.

Now assume that all elements constructed by lossy transmission lines have the same $\gamma(s)$ and $\delta(s)$, then the impedances of short and open circuited lossy stubs will have expressions as below:

$$Z_{sh}(s)=Zo_{sh}\delta(s)\tanh[\gamma(s)\tau] \tag{4a}$$

$$Z_{op}(s)=Zo_{op}\delta(s)/\tanh[\gamma(s)\tau] \tag{4b}$$

where Zo_{sh} and Zo_{op} are similar to Zo_t, and $Zo_{sh}\delta(s)$ and $Zo_{op}\delta(s)$ are frequency dependent characteristic impedances of the short and open circuited lossy stubs.

If a LYCLN only contains short and open circuited lossy stubs, then by substituting

$$Z_1=\delta(s)\tanh[\gamma(s)\tau] \tag{5a}$$

$$Z_2=\delta(s)/\tanh[\gamma(s)\tau] \tag{5b}$$

in (4), we have

$$Z_{sh}(s)=Zo_{sh}Z_1 \tag{6a}$$

$$Z_{op}(s)=Zo_{op}Z_2 \tag{6b}$$

Thus, in terms of the transformation introduced in [1], the LYCLN can be transformed to a corresponding LLLN, or Richards' transformation can be used to correspond the short and open circuited stubs to inductors and capacitors respectively if the stubs are lossless.

But a LYCLN without a finite lossy transmission line, which is usually incorperated as *distributed lossy unit element*(DLYUE), will be practically useless. However, unlike a finite lossless transmission line, the DLYUE can't be transformed by Richards' correspondence. Furthermore, it is uncertain whether the DLYUE can be transformed as the open and short circuited lossy stubs or not. In order to solve this problem, the transformation in [1] is revised and a new theorem given below is proposed, which can be employed to transform the DLYUE to a corresponding *lumped lossless unit element*(LLLUE).

Theorem: If each element of any basic unit (BU) or of its equivalent circuit produces an individual impedance equaling to the product of Z_2 and a function of Z_1/Z_2, where Z_1 and Z_2 are any physically realizable impedances, then the impedance matrix of any lossy or lossless network N constructed by these BUs, if it exists, can be transformed to the impedance matrix of a corresponding lossless network M. (The proof is omitted)

$$\tilde{Z}(\lambda)=Z/\sqrt{Z_1Z_2}=F(\lambda^2)/\lambda \tag{7a}$$

$$Z(s)=\sqrt{Z_1Z_2}\tilde{Z}=Z_2F(Z_1/Z_2) \tag{7b}$$

Where F(x) is a matrix function of x, $\tilde{Z}(\lambda)$ and Z(s) are impedance matrices of M and N with λ and s being their complex angular frequencies and being related by

$$\lambda=\sqrt{Z_1(s)/Z_2(s)} \tag{8}$$

It can be verified easily that (8) maps both right-half of the s and λ planes to each other. Certainly, this mapping is not one-to-one, but the multiple-valuedness of the inverse corresponds merely to the periodicity of Z(s) by (7b).

It should be emphasize that although this theorem is based upon the theorem in [1] and has similar form of transformation as that of the corollary 1 in [1], the conditions between them are different. The former extends the original condition of the later to even wide case. That is to say, it only needs each element in the impedance matrix or in the equivalent circuit of a BU has the form of $Z_2f(Z_1/Z_2)$, where f(x) is a function of x. Of course, this form can reduce to Z_1 or Z_2. Therefore, this theorem is especially suitable to those elements which could not simply be represented by the impedance which is proportional to Z_1 or Z_2. The advantage of the theorem can be seen clearly by the following example.

Example; A DLYUE with delay τ (shown in Fig.1) can be considered as a BU, then it can be found by substituting (5) in the elements of the equivalent circuit of the DLYUE that the condition of the theorem is satisfied, i.e.,

$$z_1=\frac{Zo_t\delta(s)}{\tanh[\gamma(s)\tau]}=Zo_tZ_2 \quad (9a)$$

$$z_2=\frac{Zo_t\delta(s)}{\sinh[\gamma(s)\tau]}=Zo_tZ_2\sqrt{1-(Z_1/Z_2)}. \quad (9b)$$

Here, Zo_t, $\delta(s)$ and $\gamma(s)$ are given as (2) and (3). For convenience of the transformation, the impedance matrix of the DLYUE,

$$Z=\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}=\begin{bmatrix} z_1+z_2 & z_2 \\ z_2 & z_1+z_2 \end{bmatrix} \quad (10)$$

is used. Thus, with the help of the transformation (7), the impedance matrix of a corresponding LLLUE can be obtained.

$$\tilde{Z}=\begin{bmatrix} \tilde{z}_{11} & \tilde{z}_{12} \\ \tilde{z}_{21} & \tilde{z}_{22} \end{bmatrix}=\frac{Z}{\sqrt{Z_1Z_2}}=\frac{1}{\lambda}\begin{bmatrix} Zo_t & Zo_t/\sqrt{1-\lambda^2} \\ Zo_t/\sqrt{1-\lambda^2} & Zo_t \end{bmatrix} \quad (11)$$

here $\lambda=\sqrt{Z_1/Z_2}=\tanh[\gamma(s)\tau]$.

It is known that the scattering parameters is very suitable for describing a transmission line and their concepts are very clear. Therefore, by means of the following transformation,

$$\tilde{S}(\lambda)=\{(I+S)-\sqrt{Z_1Z_2}(I-S)\}\{(I+S)\pm\sqrt{Z_1Z_2}(I-S)\}^{-1} \quad (12a)$$

$$S(s)=\{\sqrt{Z_1Z_2}(I+\tilde{S})-(I-\tilde{S})\}\{\sqrt{Z_1Z_2}(I+\tilde{S})+(I-\tilde{S})\}^{-1} \quad (12b)$$

where S(s) is the scattering matrix of any lossy or lossless network N, $\tilde{S}(\lambda)$ is the scattering matrix of a corresponding lossless network M, and I is the identity matrix, the familiar scattering matrix of the LLLUE can be achieved as

$$\tilde{S}(\lambda)=\begin{vmatrix} 0 & \sqrt{1-\lambda^2}/(1+\lambda) \\ \sqrt{1-\lambda^2}/(1+\lambda) & 0 \end{vmatrix} \quad (13)$$

Since a LYCLN is usually constructed by the DLYUE, open and short circuited lossy stubs, according to above description, we arrive at an important conclusion that if all elements of the LYCLN have common propagation constant $\gamma(s)$ and their characteristic impedances proportional to the frequency dependent function $\delta(s)$, the LYCLN can be transformed to a corresponding LLLN.

III. APPLICATION OF THE TRANSFORMATION THEORY

In order to clearly demonstrate the synthesis procedure of the LYCLN and its application in MMIC's, a detailed synthesis steps of a broad-band monolithic microwave integrated FET amplifier is illustrated.

Step 1: The LLLNs, which can be realized by ideal transformers, LLLUEs, lossless inductors and capacitors, are used as matching networks and are supposed to have the following forms of scattering parameters

$$\tilde{e}_{11,v}(\lambda)=\frac{h(\lambda)}{g(\lambda)}=\frac{h_1+h_2\lambda+h_3\lambda^2+\ldots+h_{n+1}\lambda^n}{g_1+g_2\lambda+g_3\lambda^2+\ldots+g_{n+1}\lambda^n} \quad (14a)$$

$$\tilde{e}_{12,v}(\lambda)=\tilde{e}_{21,v}(\lambda)=\frac{(+/-)f(\lambda)}{g(\lambda)}=\frac{(+/-)\lambda^k(1+\lambda^2)^{m/2}}{g(\lambda)} \quad (14b)$$

$$\tilde{e}_{22,v}(\lambda)=(-1)^{k+1}h(-\lambda)/g(\lambda). \quad (14c)$$

$$(v=1,2,\ldots,NM)$$

Here NM is the number of the matching networks; $h(\lambda)$ and $g(\lambda)$ are numerator and denominator polynomials of $\tilde{e}_{11,v}(\lambda)$ and have the same degree n; the numerator polynomial $f(\lambda)$ of $\tilde{e}_{12,v}(\lambda)$ has degree $m+k<n$, where m and k represent the numbers of LLLUEs and highpass elements respectively. Therefore, the number of lowpass elements are determined by n-(m+k). It should be pointed out that the delay τ in our technique is used as a variable for obtaining even better performances.

Step 2: d_L and d_C of the DLYUEs and lossy stubs are assumed to be 10^{10} Ω/H and 10^8 Ω^{-1}/F respectively. By substituting (2a) and (3a) in (5), Z_1 and Z_2 in (12) can be calculated. Therefore, the lossless scattering parameters in (14) can be transformaed by (12) to the corresponding lossy ones $e_{ij,v}(s)(i,j=1,2)$.

Step 3: The numerically specified scattering parameters of a HP 1μm MESFET are used[2] across an octave band 7-14GHz. Z_g and Z_l, the source and load impedances of the amplifier, are specified. For this example, $Z_g=Z_l=50$ Ω. The expression for the transducer power gain (TPG) of a lossy match amplifier[1] is employed in the optimization. In consideration of the losses resulted by the lossy matching elements and the minimum ilateral conjugate gain at 14GHz, $T_o(\omega)$, the flat gain level, is specified to be 7.5dB over the octave band.

Step 4: After optimization, $\tilde{e}_{11,v}$, the input reflection factor of the supposed LLLN, which correponds to a LYCLN, is obtained. The LLLN can be achieved by realizing the corresponding input impedance $(1+\tilde{e}_{11,v})/(1-\tilde{e}_{11,v})$. Afterwards, the topology of the LYCLN can be easily obtained by substituting lossy commensurate transmission lines, open and short circuited lossy stubs for the corresponding LLLUEs, lossy capacitors and inductors respectively. For this example, the supposed LLLNs for input and output matches are computered as

$$\tilde{e}_{11,1}(\lambda)=\frac{-0.4712\lambda-0.7156\lambda^2+0.7698\lambda^3}{1+2.6978\lambda+2.5281\lambda^2+0.7698\lambda^3}$$

$$\tilde{e}_{11,2}(\lambda)=\frac{1.9853\lambda+0.9648\lambda^2+0.7026\lambda^3}{1+3.1131\lambda+1.875\lambda^2+0.7026\lambda^3}$$

where τ_{in} and τ_{out}, the delays of the lines in the input and output matching networks are different and both networks have the same numbers of the matching elements and transmission lines. Their topologies can be synthesized as shown in Fig.2.

Discussion: The method described in this paper extends the real frequency technique in [5,6] to the lossy case and is traightforward in comparison with the technique presented in [2]. It can handle lossy commensurate lines and stubs with arbitrary frequency dependent propagation constants and characteristic impedances and has all of the advantages that the real frequency technique may have. Moreover, from the gain performances shown in Fig.3, it can be seen that i) the optimized lossy gain is about 0.5 dB on the average lower than the gain calculated by taking the lossy elements out or equivalently leting d_L and d_C be equal to zero. In practice, the lossy parameters, d_L and d_C, may vary in a wide range, so that it is worthy to take the losses of the elements into consideration in network synthesis. ii) the ripple of the lossless gain is only half of that of the initial response in [2].

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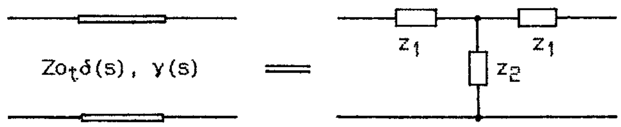

Fig. 1 The equivalent circuit of a DLYUE

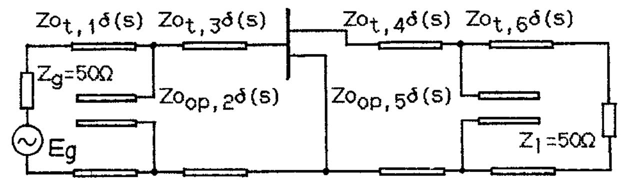

Fig.2 7-14GHz range amplifier circuit $Zo_{t,1}$=44.374Ω, $Zo_{op,2}$=38.596 Ω, $Zo_{t,3}$=66.956 Ω, $Zo_{t,4}$=170.236 Ω, $Zo_{op,5}$=205.2 Ω, $Zo_{t,6}$=84.686 Ω, τ_{in}=2.6772mm, τ_{out}=4.0912 mm, $\delta(s)=[(s+10^{10})/(s+10^8)]^{1/2}$, $\gamma(s)=[(s+10^{10})(s+10^8)]^{1/2}$

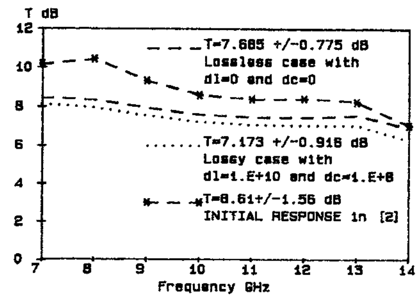

Fig.3 The frequency response of the amplifier